Volatility Smiles

- Chapter 18

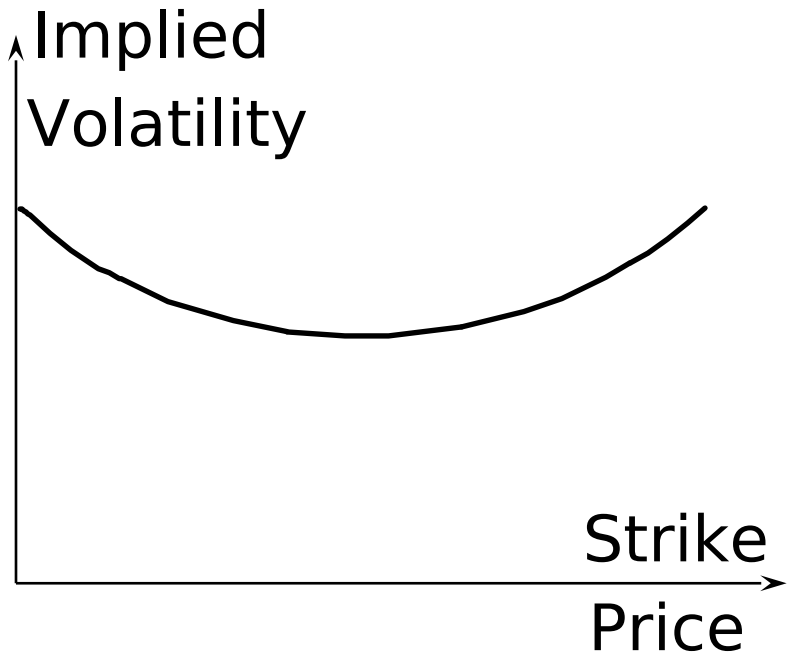
What is a Volatility Smile?

- It is the relationship between implied volatility and strike price for options with a certain maturity
- The volatility smile for European call options should be exactly the same as that for European put options
- The same is at least approximately true for American options

Why the Volatility Smile is the Same for Calls and Put

- Put-call parity $p + S_0 e^{-qT} = c + Ke^{-rT}$ holds for market prices (p_{mkt} and c_{mkt}) and for Black-Scholes prices (p_{bs} and c_{bs})
- It follows that $p_{\text{mkt}} - p_{\text{bs}} = c_{\text{mkt}} - c_{\text{bs}}$
- When $p_{\text{bs}} = p_{\text{mkt}}$, it must be true that $c_{\text{bs}} = c_{\text{mkt}}$
- It follows that the implied volatility calculated from a European call option should be the same as that calculated from a European put option when both have the same strike price and maturity

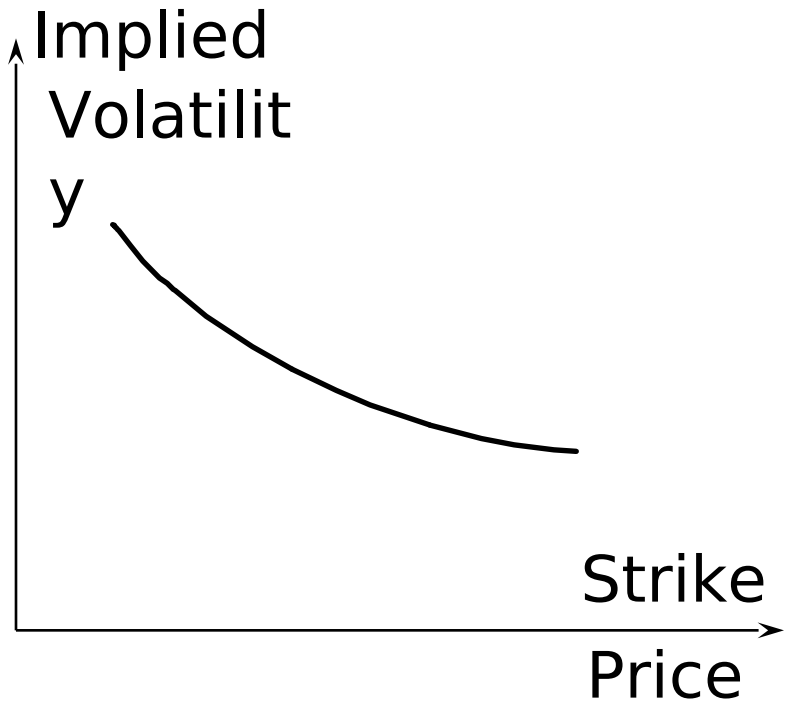
The Volatility Smile for Foreign Currency Options
(Figure 18.1, page 391)



Implied Distribution for Foreign Currency Options (Figure 18.2, page 391)

- Both tails are heavier than the lognormal distribution
- It is also “more peaked” than the lognormal distribution

The Volatility Smile for Equity Options (Figure 18.3, page 394)



Implied Distribution for Equity Options (Figure 18.4, page 394)

- The left tail is heavier and the right tail is less heavy than the lognormal distribution

Other Volatility Smiles?

What is the volatility smile if

- True distribution has a less heavy left tail and heavier right tail
- True distribution has both a less heavy left tail and a less heavy right tail

Ways of Characterizing the Volatility Smiles

- Plot implied volatility against K/S_0 (The volatility smile is then more stable)
- Plot implied volatility against K/F_0 (Traders usually define an option as at-the-money when K equals the forward price, F_0, not when it equals the spot price S_0)
- Plot implied volatility against delta of the option (This approach allows the volatility smile to be applied to some non-standard options. At-the money is defined as a call with a delta of 0.5 or a put with a delta of −0.5. These are referred to as 50-delta options)

Possible Causes of Volatility Smile

- Asset price exhibits jumps rather than continuous changes
- Volatility for asset price is stochastic
  - In the case of an exchange rate volatility is not heavily correlated with the exchange rate. The effect of a stochastic volatility is to create a symmetrical smile
  - In the case of equities volatility is negatively related to stock prices because of the impact of leverage. This is consistent with the skew that is observed in practice

Volatility Term Structure

- In addition to calculating a volatility smile, traders also calculate a volatility term structure
- This shows the variation of implied volatility with the time to maturity of the option

Volatility Term Structure

The volatility term structure tends to be downward sloping when volatility is high and upward sloping when it is low

Example of a Volatility Surface

(Table 18.2, page 397)

	K/S_0				
	0.90	0.95	1.00	1.05	1.10
1 mnth	14.2	13.0	12.0	13.1	14.5
3 mnth	14.0	13.0	12.0	13.1	14.2
6 mnth	14.1	13.3	12.5	13.4	14.3
1 year	14.7	14.0	13.5	14.0	14.8
2 year	15.0	14.4	14.0	14.5	15.1
5 year	14.8	14.6	14.4	14.7	15.0

Greek Letters

- If the Black-Scholes price, c_{BS} is expressed as a function of the stock price, S, and the implied volatility, σ_{imp}, the delta of a call is

$$\frac{\partial c_{\mathrm{BS}}}{\partial S}+\frac{\partial c_{\mathrm{BS}}}{\partial \sigma_{\mathrm{imp}}}\frac{\partial \sigma_{\mathrm{imp}}}{\partial S}$$

- Is the delta higher or lower than $\dfrac{\partial c_{\mathrm{BS}}}{\partial S}$?